$\neg(Q \Rightarrow R) \Leftrightarrow (\neg R \wedge Q)$ is a tautology

First, we establish, with a calculation, that $\neg(Q \Rightarrow R) \stackrel{val}{==} (\neg R \wedge Q)$:

$$\neg(Q \Rightarrow R)$$

$\stackrel{val}{==}$ { Implication }

$$\neg(\neg Q \vee R)$$

$\stackrel{val}{==}$ { De Morgan }

$$\neg\neg Q \wedge \neg R$$

$\stackrel{val}{==}$ { Double negation }

$$\neg R \wedge Q$$

From $\neg(Q \Rightarrow R) \stackrel{val}{==} \neg R \wedge Q$ it follows that $\neg(Q \Rightarrow R) \Leftrightarrow (\neg R \wedge Q)$ is a tautology.

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$\overset{val}{=\!=}$ { Implication }

$$\neg(\boxed{\neg Q \vee R})$$

$\overset{val}{=\!=}$ { De Morgan }

$$\neg\neg Q \wedge \neg R$$

$\overset{val}{=\!=}$ { Double negation }

$$\neg R \wedge Q$$

Explanation:

Substituting Q for P and R for Q in

$P \Rightarrow Q \overset{val}{=\!=} \neg P \vee Q$ (Implication)

we get, by the substitution rule:

$Q \Rightarrow R \overset{val}{=\!=} \neg Q \vee R$.

(The application of this equivalence in the calculation involves an application of Leibniz.)

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$\neg(Q \Rightarrow R)$

$\stackrel{val}{=\!=}$ { Implication }

$\neg(\neg Q \vee R)$

$\stackrel{val}{=\!=}$ { De Morgan }

$\neg\neg Q \wedge \neg R$

$\stackrel{val}{=\!=}$ { Double negation }

$\neg R \wedge Q$

Explanation:

Substituting $\neg Q$ for P and R for Q in

$\neg(P \vee Q) \stackrel{val}{=\!=} \neg P \wedge \neg Q$ (De Morgan)

we get, by the substitution rule:

$\neg(\neg Q \vee R) \stackrel{val}{=\!=} \neg\neg Q \wedge \neg R$.

From $\neg(Q \Rightarrow R) \stackrel{val}{=\!=} \neg R \wedge Q$ it follows that $\neg(Q \Rightarrow R) \Leftrightarrow (\neg R \wedge Q)$ is a tautology.

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$\neg(Q \Rightarrow R)$

$\stackrel{val}{==}$ { Implication }

$\neg(\neg Q \vee R)$

$\stackrel{val}{==}$ { De Morgan }

$\boxed{\neg\neg Q} \wedge \neg R$

$\stackrel{val}{==}$ { Double negation }

$\neg R \wedge \boxed{Q}$

Explanation:

Substituting Q for P in

$\neg\neg P \stackrel{val}{==} P$ (Double negation)

we get, by the substitution rule:

$\neg\neg Q \stackrel{val}{==} Q$.

(The application of this equivalence in the calculation involves an application of Leibniz, and is followed by an application of Commutativity.)

From $\neg(Q \Rightarrow R) \stackrel{val}{==} \neg R \wedge Q$ it follows that $\neg(Q \Rightarrow R) \Leftrightarrow (\neg R \wedge Q)$ is a tautology.

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From $\neg(Q \Rightarrow R) \overset{val}{=\!=} \neg R \wedge Q$ it follows that $\neg(Q \Rightarrow R) \Leftrightarrow (\neg R \wedge Q)$ is a tautology.