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For instance, using the abbreviation $P(n)$ we can now reformulate the original goal $\forall_n[n \in \mathbb{N} : a_n = 2^n + 1]$ as $\forall_n[n \in \mathbb{N} : P(n)]$.

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It follows, according to the definition of P, that $P(i+1)$ indeed holds.

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On the next slide we will also give and discuss also a textual version of the proof.

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$$\begin{array}{lcl} a_0 & := & 2 \\ a_{i+1} & := & 2a_i - 1 \qquad (\text{for all } i \in \mathbb{N}) \enspace . \end{array}$$

Prove that $a_n = 2^n + 1$ for all $n \in \mathbb{N}$.

We prove by induction on $n \in \mathbb{N}$ that $a_n = 2^n + 1$.

(Basis) Note that, according to the definition of the sequence $a_0, a_1, a_2, \ldots$, $a_0 = 2$ and $2 = 2^0 + 1$, so, indeed, $a_0 = 2^0 + 1$.

(Step) Let $i \in \mathbb{N}$, and suppose that $a_i = 2^i + 1$ (induction hypothesis). Then

$$\begin{array}{lll} a_{i+1} & = 2a_i - 1 & (\text{by def. } a_0, a_1, a_2, \ldots) \\ & = 2(2^i + 1) - 1 & (\text{by the induction hypothesis}) \\ & = 2^{i+1} + 1 & (\text{by elementary mathematics}) \enspace . \end{array}$$

It should be noted that the above proof contains very little redundancy. In particular, (also) if you give a textual proof by induction you should

1. state that your proof is by induction;
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Define the unary predicate P on $\mathbb{N}$ by

$$P(n) := [a_n = 2^n + 1] \enspace .$$

We have the following derivation:

$a_0 = 2 = 2^0 + 1$

{ Def. P }

(B) $P(0)$

var i; $i \in \mathbb{N}$

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(S) $\forall_i[i \in \mathbb{N} : P(i) \Rightarrow P(i+1)]$

{ Induction on (B) and (S): }

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