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(6) Case $p \geq 11$: $7 < p - 3 < p$, so ($\forall$-elim) $P(p - 3)$.

Pick k, ℓ with $k, \ell \in \mathbb{N}$ and $p - 3 = k \cdot 3 + \ell \cdot 5$

(7) Then : $p = (p - 3) + 3$
$= (k \cdot 3 + \ell \cdot 5) + 3$
$= (k + 1) \cdot 3 + \ell \cdot 5$

So ($\exists^*$-intro) $P(p)$

(8) { Case distinction:

$p \in \mathbb{N} \wedge p > 7 \stackrel{val}{==} p = 8 \vee p = 9 \vee p = 10 \vee p \geq 11$ }

(9) $P(p)$

(10) $\forall_j[j \in \mathbb{N} \wedge 7 < j < p : P(j)] \Rightarrow P(p)$

(11) $\forall_p[p \in \mathbb{N} \wedge p > 7 : \forall_j[j \in \mathbb{N} \wedge 7 < j < p : P(j)] \Rightarrow P(p)]$

(12) { Strong induction: }

(13) $\forall_p[p \in \mathbb{N} \wedge p > 7 : P(p)]$

We prove with strong induction on p that every postage p greater than 7 can be formed using only 3-cent and 5-cent stamps.
Let p be an arbitrary postage > 7.
Suppose: every postage p' with $7 < p' < p$ can be formed using only 3-cent and 5-cent stamps (IH).
We now distinguish four cases:

- If $p = 8$, then p can be formed with one 3-cent stamp, and one 5-cent stamp.
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Thereby the result is proved.

Define P on $\mathbb{N}$ by:

$$P(p) := \exists_{k,\ell}[k, \ell \in \mathbb{N} : p = k{\cdot}3{+}\ell{\cdot}5] \ .$$

(1) $\boxed{\textbf{var } p;\, p \in \mathbb{N} \wedge p > 7}$

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