TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Final examination Logic & Set Theory (2IT61/2IT07)

Thursday October 31, 2013, 9:00–12:00 hrs.

You are **not** allowed to use any books, notes, or other course material. Your solutions to the problems have to be formulated and written down in a clear and precise manner.

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(1) 1. Determine $\mathcal{P}(\{0,1\}) \times \mathcal{P}(\emptyset)$.

(2) 2. Prove that the formulas

$$P \Rightarrow ((Q \Rightarrow R) \wedge (Q \vee R)) \quad \text{and} \quad (\neg P \Rightarrow Q) \Rightarrow R$$

are comparable (i.e., the left-hand side formula is stronger than the right-hand side formula, or vice versa).

3. Let P be the set of all people, let Anna denote a particular person in P, and let B be the set of all books. Furthermore, let O be a predicate on $P \times B$, and let L be a predicate on $P \times B \times P$, with the following interpretations:

$O(p, b)$ means 'p owns b', and
$L(p, b, q)$ means 'p has lent out b to q'.

Give formulas of predicate logic that express the following statements:

(1) (a) Everybody owns a book.
(1) (b) Anna has only lent out books she owns.

(2) 4. Prove with a *derivation* (i.e., using the methods described in *Part II* of the book) that the formula

$$\exists_x \forall_y[\neg P(y) : Q(y,x)] \Rightarrow \forall_z[P(z) \vee \exists_u[Q(z,u)]]$$

is a tautology.

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(3) 5. Prove that the following formula holds for all sets A, B and C:

$$B \subseteq A \;\Rightarrow\; B \subseteq (C \backslash A)^{\mathrm{C}} \;.$$

6. Consider the mapping $f : \mathbb{R} \to \mathbb{R}$ that is defined, for all $x \in \mathbb{R}$, by

$$f(x) = x^2 - 4x + 4 \;.$$

(2) (a) Determine $f^{\leftarrow}(\{1\})$.

(1) (b) Give the formula that expresses 'f is an injection' and show with a counterexample that f is not an injection.

(3) 7. Prove by induction that $n^3 - n$ is divisible by 3 for all $n \in \mathbb{N}$.

8. Define the relation R on $\mathbb{N} \times \mathbb{N}$ by

$$(x_1, y_1)\; R\; (x_2, y_2) \text{ if, and only if, } x_2 - x_1 < y_2 - y_1 \;.$$

(2) (a) Prove that $\langle \mathbb{N} \times \mathbb{N}, R \rangle$ is an *irreflexive ordering.*

(1) (b) Draw a Hasse diagram of $\langle \{0,1\} \times \{0,1\}, R \rangle$.

(1) (c) Give the maximal and minimal elements of the set $\{0,1\} \times \{0,1\}$ in the irreflexive ordering $\langle \mathbb{N} \times \mathbb{N}, R \rangle$.

The number between parentheses in front of a problem indicates how many points you score with a correct answer to it. A partially correct answer is sometimes awarded with a fraction of those points. The grade for this examination will be determined by dividing the total number of scored points by 2.