

**Final examination Logic & Set Theory (2IT61/2IT07)**

Thursday January 23, 2014, 14:00–17:00 hrs.

You are **not** allowed to use any books, notes, or other course material. Your solutions to the problems have to be formulated and written down in a clear and precise manner.

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- (2) 1. Prove that the formulas

$$(P \Rightarrow Q) \vee R \quad \text{and} \quad P \vee \neg(Q \vee R)$$

are incomparable.

- (2) 2. Prove with a *calculation* (i.e., using the methods described in Part 1 of the book) that

$$P \Rightarrow (Q \Rightarrow R) \stackrel{val}{=} (Q \vee R) \Rightarrow (P \Rightarrow R).$$

3. Write the following sentences as a formula of predicate logic (for (b), you may use  $\mathbb{P}$  to denote the set of all primes):

- (1) (a) There is no natural between 3 and 7 that is a square.  
(1) (b) Every odd natural can be written as the sum of the squares of at most three primes.

- (2) 4. Give three different elements of the set  $\mathcal{P}(\{3, 7\} \times \mathbb{N} \times \{\emptyset\})$ .

- (3) 5. Let  $F : A \rightarrow B$  be an injective mapping. Prove with a *derivation* (i.e., using the methods described in *Part II* and *III* of the book) that

$$\forall_{X,Y}[X, Y \subseteq A : X \cap Y = \emptyset \Rightarrow F(X) \cap F(Y) = \emptyset].$$

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- (4) 6. The sequence  $a_0, a_1, a_2, \dots$  is inductively defined by

$$\begin{aligned} a_0 &:= 4 \\ a_{n+1} &:= 3 \cdot a_n - 2 \cdot n - 5 \end{aligned}$$

Prove that  $a_n = 3^n + n + 3$  for all  $n \in \mathbb{N}$ .

7. Define the relation  $R$  on  $\mathbb{N} \times \mathbb{N}$  by

$(a, b) R (x, y)$  if, and only if,  $a < x \wedge b \geq y$  (for  $a, b, x, y \in \mathbb{N}$ ) .

- (2) (a) Prove that  $\langle \mathbb{N} \times \mathbb{N}, R \rangle$  is an *irreflexive ordering*.  
(2) (b) Draw a Hasse diagram of  $\langle \{2, 3\} \times \{5, 6, 7\}, R \rangle$ .  
(1) (c) Give the minimal elements of  $\langle \mathbb{N} \times \mathbb{N}, R \rangle$ .

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The number between parentheses in front of a problem indicates how many points you score with a correct answer to it. A partially correct answer is sometimes awarded with a fraction of those points. The grade for this examination will be determined by dividing the total number of scored points by 2.