TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Final examination Logic & Set Theory (2IT61/2IT07)

Monday April 7, 2014, 14:00–17:00 hrs.

You are **not** allowed to use any books, notes, or other course material. Your solutions to the problems have to be formulated and written down in a clear and precise manner.

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(1) 1. Show that the following abstract proposition is contingent (i.e., neither a tautology, nor a contradiction):

$$(a \Rightarrow \neg b) \vee \neg(\neg c \wedge d) \vee ((d \wedge e) \Leftrightarrow \texttt{False}) \ .$$

(2) 2. Prove with a *calculation* (i.e., using the methods described in *Part I* of the book) that

$$(P \Rightarrow \neg Q) \wedge (\neg(P \wedge R) \vee Q) \stackrel{val}{==} P \Rightarrow \neg(Q \vee R) \ .$$

3. Let **P** be the set of all people, and let M be a binary predicate on **P** with the following interpretation:

$$M(x, y) : \text{`}x \text{ is married to } y\text{'} \ .$$

Give formulas of predicate logic that express the following statements:

(1) (a) Not everybody is married.

(1) (b) Everybody has at most one spouse.
(Hints: Use that y is a spouse of x if, and only if, x is married to y, and write $y = z$ to express that y and z denote the same person.)

(2) 4. Prove with a *derivation* (i.e., using the methods described in *Part II* of the book) that the formula

$$(\forall_x[P(x) : \neg Q(x)] \wedge \neg\exists_y[R(y)]) \Rightarrow \neg\exists_z[P(z) : Q(z) \vee R(z)]$$

is a tautology.

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(3) 5. Prove that the following formula holds for all sets A, B and C:

$$A \backslash C \subseteq B^{\mathrm{C}} \;\Rightarrow\; A \cap B \subseteq C \ .$$

6. Consider the mapping $F : \mathbb{N}^2 \to \mathbb{N}$ that is defined, for all $x, y \in \mathbb{N}$, by

$$F((x,y)) = x + y \ .$$

(2) (a) Determine $F^{\leftarrow}(\{2,3\})$.

(1) (b) Is F a bijection? (Motivate your answer!)

(4) 7. The infinite sequence of natural numbers $a_0, a_1, a_2, \ldots$ is inductively defined by

$$\begin{aligned} a_0 &:= 3 \\ a_{i+1} &:= 2a_i - 2 \qquad (i \in \mathbb{N}) \ . \end{aligned}$$

Prove that $a_n = 2^n + 2$ for all $n \in \mathbb{N}$.

8. Define the relation R on $\mathbb{N}^+ \times \mathbb{N}^+$ by

$$(x_1, y_1)\ R\ (x_2, y_2) \text{ if, and only if,}$$
$$c^2 \cdot (x_1 + y_1) = x_2 + y_2 \text{ for some } c \in \mathbb{N}^+ \text{ with } c \geq 2 \ .$$

(Recall that $\mathbb{N}^+ = \{n \in \mathbb{N} \mid n > 0\}$.)

(2) (a) Prove that $\langle \mathbb{N}^+ \times \mathbb{N}^+, R\rangle$ is an *irreflexive ordering*.

(1) (b) Show with a counterexample that $\langle \mathbb{N}^+ \times \mathbb{N}^+, R\rangle$ is not linear.

The number between parentheses in front of a problem indicates how many points you score with a correct answer to it. A partially correct answer is sometimes awarded with a fraction of those points. The grade for this examination will be determined by dividing the total number of scored points by 2.