TU/e Technische Universiteit Eindhoven University of Technology

Student name:

Student number:

# Examination cover sheet

(to be completed by the examiner)

Course name: Logic and Set Theory (final examination) Course code: 2IT61/2IT07/2IHT10

Date: October 29, 2015

Start time: 9:00 End time : 12:00

Number of pages: 2

Number of questions: 7

Maximum number of points/distribution of points over questions:20 (the number between parentheses in front of a problem indicates how many points you score with a correct answer to it)

Method of determining final grade: the grade for this examination will be determined by dividing the total number of scored points by 2

Answering style: open questions

Exam inspection: a review session will be organised

Other remarks: A partially correct answer is sometimes awarded with a fraction of the points.

## Instructions for students and invigilators

### Permitted examination aids (to be supplied by students):

- ☐ Notebook
- ☐ Calculator
- ☐ Graphic calculator
- ☐ Lecture notes/book
- ☐ One A4 sheet of annotations
- ☐ Dictionar(y)(ies). If yes, please specify:
- ☐ Other:

**Important:**

- examinees are only permitted to visit the toilets under supervision
- it is not permitted to leave the examination room within 15 minutes of the start and within the final 15 minutes of the examination, unless stated otherwise
- examination scripts (fully completed examination paper, stating name, student number, etc.) must always be handed in
- the house rules must be observed during the examination
- the instructions of examiners and invigilators must be followed
- no pencil cases are permitted on desks
- examinees are not permitted to share examination aids or lend them to each other

**During written examinations, the following actions will in any case be deemed to constitute fraud or attempted fraud:**

- using another person's proof of identity/campus card (student identity card)
- having a mobile telephone or any other type of media-carrying device on your desk or in your clothes
- using, or attempting to use, unauthorized resources and aids, such as the internet, a mobile telephone, etc.
- using a clicker that does not belong to you
- having any paper at hand other than that provided by TU/e, unless stated otherwise
- visiting the toilet (or going outside) without permission or supervision

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(2) 1. Determine whether the abstract propositions

$$(a \wedge \neg b) \Rightarrow b \quad \text{and} \quad a \wedge (\neg b \Rightarrow b)$$

are *comparable* (i.e., the abstract proposition on the left is stronger than the abstract proposition on the right, or vice versa). Motivate your answer with a proof or a counterexample.

2. Let **P** be the set of all people; we assume that Anna and Bert are people (i.e., particular elements of the set **P**). Let $M$ and $F$ be unary predicates on **P** and let $C$ and $Y$ be binary predicates on **P**, with the following interpretations:

$M(x)$ means $x$ *is male*,
$F(x)$ means $x$ *is female*,
$C(x, y)$ means $x$ *is a child of* $y$,
$Y(x, y)$ means $x$ *is younger than* $y$.

Give formulas of predicate logic that express the following statements:

(1) (a) Everybody has a father.
(1) (b) Anna is a younger sister of Bert.

(3) 3. Prove with a *derivation* (i.e., using the methods described in *Part II* of the book) that the formula

$$\exists_x \forall_y [P(y) \vee Q(y, x)] \Rightarrow \forall_z [\neg P(z) : \exists_u [Q(z, u)]]$$

is a tautology.

(2) 4. Prove that $(x < 0 \vee x > 2) \Rightarrow x^2 - 2x > 0$ for all $x \in \mathbb{R}$.

(3) 5. Let the sequence $a_0, a_1, a_2, \ldots$ be inductively defined by

$$a_0 := 3$$
$$a_1 := 5$$
$$a_{i+2} := 4a_{i+1} - 3a_i \qquad (i \in \mathbb{N}).$$

Prove that $a_n = 3^n + 2$ for all $n \in \mathbb{N}$.

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(2) 6. (a) Show with a counterexample that the formula

$$\forall_{X,Y}[X, Y \subseteq A : F(X) \subseteq F(Y) \Rightarrow X \backslash Y = \emptyset]$$

does not hold for all mappings $F : A \to B$.

(2) (b) Prove that if $F : A \to B$ is an injection, then

$$\forall_{X,Y}[X, Y \subseteq A : F(X) \subseteq F(Y) \Rightarrow X \backslash Y = \emptyset] \ .$$

7. We define a binary relation $R$ on $\mathbb{N}^+$, the set of all positive natural numbers, by

$$k\ R\ \ell \text{ if, and only if, there exists } c \in \mathbb{N} \text{ with } c \geq 2 \text{ such that } \ell = c \cdot k.$$

(2) (a) Prove that $R$ is transitive.

Let $V = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

(1) (b) Make a Hasse diagram of $\langle V, R \rangle$.

(1) (c) What are the minimal elements of $V$ in $\langle V, R \rangle$?
What are the maximal elements of $V$ in $\langle V, R \rangle$?