TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Final examination Logic & Set Theory (2IT61/2IT07/2IHT10)

Thursday January 21, 2016, 13:30–16:30 hrs.

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(3) 1. Indicate which of the following expressions are and which are not propositions:

(a) Roses are blue

(b) $a \wedge b \Rightarrow c$

(c) $\forall x[x \in \mathbb{R} : x + y > 3]$

(d) $\forall i[i \in \mathbb{N} : P(i) \Rightarrow P(i+1)] \Rightarrow \forall i[i \in \mathbb{N} : P(i)]$

(e) $\exists x, y[x, y \in \mathbb{Z} : x^2 > 3 \wedge x + y]$

(3) 2. Show with a calculation that $(R \wedge Q) \vee (P \wedge \neg Q)$ and $Q \Rightarrow R$ are comparable.

3. Determine whether the following formulas hold for all sets A, B and C. If so, give a proof, if not, give a counterexample.

(2) (a) $((A \backslash B) \cap C = \emptyset) \Rightarrow (C \cap (A \cup B) = \emptyset)$

(2) (b) $(C \cap (A \cup B) = \emptyset) \Rightarrow ((A \backslash B) \cap C = \emptyset)$

4. Define the relation R on $\mathbb{N} \times \mathbb{N}$ by

$$(a, b)R(c, d) \text{ iff } ab < cd.$$

(3) (a) Prove that R is an ordering.

(2) (b) Draw a Hasse-diagram of $\langle \{0, 1, 2\} \times \{0, 1, 2\}, R \rangle$.

(1) (c) Give the minimal and maximal elements of the subset

$$\{(0, 1), (1, 0), (1, 1), (2, 0), (2, 1)\}.$$

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(5) 5. Define the sequence $a_0, a_1, a_2, \ldots$ as follows:

$$\begin{array}{rcll} a_0 & := & 2 & \\ a_{n+1} & := & 5a_n - 4 & \qquad (n \in \mathbb{N}) \end{array}$$

Prove that $\forall n[n \in \mathbb{N} : a_n = 5^n + 1]$.

6. Consider the mapping $f : \mathbb{N} \to \mathbb{N}$ that is defined by $f(q) = 2q + 3$, for all $q \in \mathbb{N}$.

(2) (a) If f is injective, prove it, if it is not, show this by means of a counterexample.

(2) (b) If f is surjective, prove it, if it is not, show this by means of a counterexample.

(2) (c) Determine $\mathcal{P}(f^{\leftarrow}(f(\{2, 3, 4\})))$.

(3) 7. Prove with a derivation (i.e., using only the methods described in part II of the book) that the formula

$$\forall y[P(y) : R(y)] \Rightarrow (\exists u[\neg P(u) \Rightarrow Q(u)] \Rightarrow \exists q[Q(q) \vee R(q)])$$

is a tautology.

The number between parentheses in front of a problem indicates how many points you score with a correct answer to it. A partially correct answer is sometimes awarded with a fraction of those points. The grade for this examination will be determined by dividing the total number of scored points by 3.