TU/e Technische Universiteit Eindhoven University of Technology

Student name:

Student number:

Examination cover sheet

(to be completed by the examiner)

Course name: Logic and Set Theory (final examination) | Course code: 2IT61

Date: November 3, 2016

Start time: 9:00 | End time : 12:00

Number of pages: 2

Number of questions: 7

Maximum number of points/distribution of points over questions: 20 (the number between parentheses in front of a problem indicates how many points you score with a correct answer to it)

Method of determining final grade: the grade for this examination will be determined by dividing the total number of scored points by 2

Answering style: open questions

Exam inspection: a review session will be organised

Other remarks: A partially correct answer is sometimes awarded with a fraction of the points.

Instructions for students and invigilators

Permitted examination aids (to be supplied by students):

- ☐ Notebook
- ☐ Calculator
- ☐ Graphic calculator
- ☐ Lecture notes/book
- ☐ One A4 sheet of annotations
- ☐ Dictionar(y)(ies). If yes, please specify:
- ☐ Other:

Important:

- examinees are only permitted to visit the toilets under supervision
- it is not permitted to leave the examination room within 15 minutes of the start and within the final 15 minutes of the examination, unless stated otherwise
- examination scripts (fully completed examination paper, stating name, student number, etc.) must always be handed in
- the house rules must be observed during the examination
- the instructions of examiners and invigilators must be followed
- no pencil cases are permitted on desks
- examinees are not permitted to share examination aids or lend them to each other

During written examinations, the following actions will in any case be deemed to constitute fraud or attempted fraud:

- using another person's proof of identity/campus card (student identity card)
- having a mobile telephone or any other type of media-carrying device on your desk or in your clothes
- using, or attempting to use, unauthorized resources and aids, such as the internet, a mobile telephone, etc.
- using a clicker that does not belong to you
- having any paper at hand other than that provided by TU/e, unless stated otherwise
- visiting the toilet (or going outside) without permission or supervision

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(2) 1. Determine whether the abstract propositions

$$\neg a \Leftrightarrow (a \vee b) \quad \text{and} \quad \neg a \wedge b$$

are *comparable* (i.e., the abstract proposition on the left is stronger than the abstract proposition on the right, or vice versa). Motivate your answer with a proof or a counterexample.

(2) 2. Give a formula of predicate logic that expresses the following sentence:

Every even natural number greater than 4 is the sum of two prime numbers.

(You may use $\mathbb{P}$ to denote the set of all prime numbers.)

(3) 3. Prove with a *derivation* (i.e., using the methods described in *Part II* of the book) that the formula

$$\exists_x[x \in \mathbb{N} : P(x) \vee Q(x)] \Rightarrow (\forall_y[y \in \mathbb{N} : \neg P(y)] \Rightarrow \exists_z[z \in \mathbb{N} : \neg Q(z) \Rightarrow R(z)])$$

is a tautology.

(2) 4. Prove that $|x - 2| \geq -x + 1$ for all $x \in \mathbb{R}$.

(3) 5. Prove that every integer postage greater than 11 can be formed using only 3-cent and 7-cent stamps.

(1) 6. (a) Show with a counterexample that the formula

$$\forall_Y[Y \in \mathcal{P}(\mathbb{N}) : \exists_X[X \in \mathcal{P}(\mathbb{Z}) : Y \subseteq F(X)]]$$

does not hold for all mappings $F : \mathbb{Z} \to \mathbb{N}$.

(1) (b) For a mapping $F : \mathbb{Z} \to \mathbb{N}$ and a set $Y \subseteq \mathbb{N}$, give the definition of the set $F^{\leftarrow}(Y)$, and prove that $F^{\leftarrow}(Y) \in \mathcal{P}(\mathbb{Z})$.

(2) (c) Prove that if $F : \mathbb{Z} \to \mathbb{N}$ is a surjection, then

$$\forall_Y[Y \in \mathcal{P}(\mathbb{N}) : \exists_X[X \in \mathcal{P}(\mathbb{Z}) : Y \subseteq F(X)]] \ .$$

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7. We define a binary relation R on $\mathbb{N}^+$, the set of all positive natural numbers, by

$$k \mathrel{R} \ell \text{ if, and only if, there exists } c \in \mathbb{N}^+ \text{ such that } c \text{ is a multiple of 3 and } \ell = c \cdot k.$$

(2) (a) Prove that R is transitive.

Let $V = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

(1) (b) Make a Hasse diagram of $\langle V, R \rangle$.

(1) (c) Define an *infinite* set $W \subseteq \mathbb{N}^+$ such that $\langle W, R \rangle$ is linear.