TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Final examination Logic & Set Theory (2IT61)

Thursday January 26, 2017, 13:30–16:30 hrs.

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(2) 1. Prove by means of a calculation that the formulas

$$\forall_z[P(z) \wedge Q(z) : \neg R(z)] \qquad \text{and} \qquad \neg\exists_u[P(u) \wedge R(u) : Q(u)]$$

are equivalent, or explain why they aren't.

(3) 2. Give a flag-style derivation (using only methods of part II of the book) that shows that the formula $(Q \Rightarrow P) \Rightarrow (\neg Q \vee (\neg P \Rightarrow R))$ is a tautology.

3. Prove or give a counterexample for the following statements:

(2) (a) For all subsets A, B, C of $\mathbb{Z}$: $(A \cap C) \cup (B \setminus (A \cup C)) \subseteq (A \cap C) \cup (B \setminus C)$.

(2) (b) For all subsets A, B, C of $\mathbb{Z}$: $(A \cap C) \cup (B \setminus C) \subseteq (A \cap C) \cup (B \setminus (A \cup C))$.

4. Define the mapping $f : \mathbb{N} \to \mathbb{N}$ inductively as follows:

$$\begin{array}{rcll} f(0) & := & 3 & \\ f(n+1) & := & 3 \cdot f(n) - 5 & \qquad (n \in \mathbb{N}) \end{array}$$

(1) (a) Determine $f(\{0, 1, 2\})$.

(4) (b) Prove that $\forall_n[n \in \mathbb{N} : f(n) = \frac{1}{2} \cdot 3^n + 2\frac{1}{2}]$.

(2) (c) Prove that f is not a bijection.

5. Define the relation R on $\mathbb{N} \times \mathbb{N}$ by

$$(x, y)R(a, b) \text{ if and only if } x < a \wedge y < a \wedge y < b \qquad (x, y, a, b \in \mathbb{N}).$$

(2) (a) Prove that R is an irreflexive ordering.

(1) (b) Draw a Hasse-diagram of $\langle \{1, 2, 3\} \times \{1, 2, 3\}, R \rangle$.

(1) (c) Give the minimal and maximal elements of $\{1, 2, 3\} \times \{1, 2, 3\}$.

The number between parentheses in front of a problem indicates how many points you score with a correct answer to it. A partially correct answer is sometimes awarded with a fraction of those points. The grade for this examination will be determined by dividing the total number of scored points by 2.