Other Binders

Lecture 3 (Chapter 10)

Slides were not discussed in the lecture.
Please read Chapter 10 of the book yourself.

Predicates and abstract function values

- $\underbrace{6m + 9n > 3}_{sentence}$ is a binary predicate

  (we get a propostion by filling out values for m and n)
- $\underbrace{6m + 9n}_{noun}$ is an abstract function value

  (we get an *object* by filling out values for m and n)

Sets

Notations for sets:

- $\{0, 1, 2, 3\} = \{3, 1, 0, 2\} = \{3, 1, 1, 1, 0, 0, 2\}$.
- $\{6\}$ is the singleton set (NB: $\{6\} \neq 6$).
- $\emptyset$ denotes the empty set (i.e., the set without elements).
- $\mathbb{N}$, $\mathbb{Z}$: standard set-theoretic notations for sets of numbers.

Describing sets using predicates:

$$\{m \in \mathbb{Z} \mid -2 \leq m \leq 1 \vee m > 3\}$$

is

"the set of all integers that are either greater or equal -2 and less or equal 1 or greater than 3"

(Alternative notation: $\text{Set}\ m : m \in \mathbb{Z} \wedge (-2 \leq m \leq 1 \vee m > 3) : m$)

Sum and Product (binders)

Sum:

$$\sum_{k=0}^{5} x_k = \sum_{k \in \{0,1,2,3,4,5\}} x_k = x_0 + x_1 + x_2 + x_3 + x_4 + x_5 \ .$$

(Alternative notation: $(\sum k : 0 \leq k < 6 : x_k)$)

Product:

$$\prod_{k=2}^{5} k = 2 \cdot 3 \cdot 4 \cdot 5 = 120 \ .$$

(Alternative notation: $\prod k : 2 \leq k \leq 5 : k$)

Counting the number of elements of a set

$\#\{0, 1, 2, 3\} = 4$

$\#\{i \in \mathbb{Z} \mid 0 < i \leq 100 \land i^2 < 90\} = 9$

(Alternative notation: $\# i : i \in \mathbb{Z} : 0 < i \leq 100 \land i^2 < 90$)

Technische Universiteit Eindhoven University of Technology